Physics AC Currents

1 Mark Questions

1.A reactive element in an AC circuit causes the current flowing

(*i*) to lead in phase by $\pi/2$

(*ii*) to lag in phase by $\pi/2$

w.r.t. the applied voltage. Identify the element in each case. **[Delhi 2010C]**

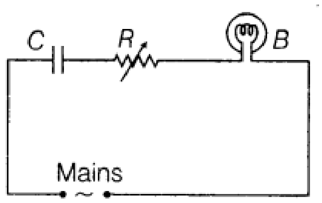
Ans

(*i*) In case of pure capacitive circuit, the current leads in phase by $\pi/2$ with respect to the applied voltage. So, the element will be a capacitor. **(1/2)**

(*ii*) In case of pure inductive circuit, the current lags in phase by $\pi/2$ with respect to the applied voltage. So, the element will be an inductor. **(1/2)**

2 Marks Questions

2.A capacitor C, a variable resistor R and a bulb B are connected in series to the AC mains in the circuit as shown. The bulb glows with some brightness. How will the glow of the bulb change if (i) a dielectric slab is introduced between the plates of the capacitor keeping resistance R to be the same (ii) the resistance R is increased keeping the same capacitance? [Delhi 2014]



Ans.

(*i*) As, the dielectric slab is introduced between the plates of the capacitor, its capacitance will increase. Hence, the potential drop across the capacitor will decrease, i.e. $V = \frac{Q}{C}$. As a result, the potential drop across the bulb will increase as they are connected in series.Thus, its brightness will increase. **(1)**

(*ii*) As the resistance R is increased, the potential drop across the resistor will increase. As a result, the potential drop across the bulb will decrease as they are connected in series. Thus, its brightness will decrease. **(1)**

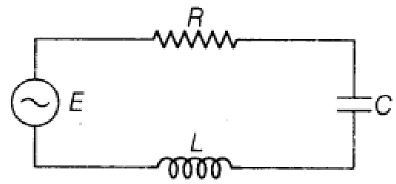
3. The figure shows a series *L-C-R* circuit connected to a variable frequency 200 V source with $L = 50$ mH, $C = 80$ μF and $R = 40\,\Omega$.

Determine

(*i*) the source frequency which derives the circuit in resonance.

(*ii*) the quality factor (Q) of the circuit.

[All India 2014C]



Ans.

Given, $L = 50\text{mH} = 50 \times 10^{-3}\text{H}$

$C = 80\,\mu\text{F} = 80 \times 10^{-6}\text{F}$

$R = 400\pi$, $V = 200$ V

(*i*) In the *L-C-R*, the resonant angular frequency is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50 \times 10^{-3} \times 80 \times 10^{-6}}}$$

$$= 500\text{rad/s}$$

$\therefore$ The actual/source frequency is given by

$$\omega = 2\pi\nu \Rightarrow \nu = \frac{\omega}{2\pi}$$

$$\Rightarrow \nu = \frac{500}{2\pi} = \frac{250}{\pi} = 79.61 \approx 80\,\text{Hz}$$

(*ii*) Quality factor, $Q = \frac{\omega_0 L}{R}$

$$= \frac{500 \times 50 \times 10^{-3}}{40} = 0.625$$

(1/2)

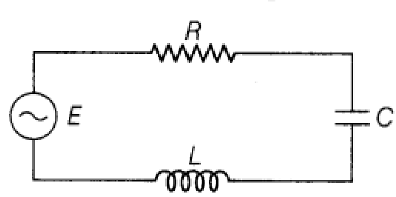
4. The figure shows a series *L-C-R* circuit connected to a variable frequency 250 V source with $L = 40$mH, $C = 100$ μF and $R = 50\,\Omega$.

Determine

(*i*) the source frequency which derives the circuit in resonance,

(*ii*) the quality factor (Q) of the circuit.

[All India 2014]



Ans.Refer to ans3

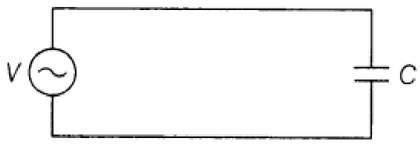
5. The figure shows a series L-C-R circuit connected to a variable frequency 220 V source with $L = 80$ mH, $C = 50\ \mu$F and and $R = 60\ \Omega$.

Determine

(*i*) the source frequency which derives the circuit in resonance.

(*ii*) the quality factor Q of the circuit.

[All India 2014]

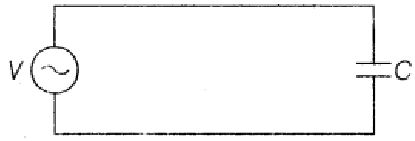


Ans Refer to ans3.

6. Show that the current leads the voltage in phase by $\pi/2$ in an AC circuit containing an ideal capacitor. **[Foreign 2014]**

Ans.

Let us consider a capacitor C connected to an AC source as shown below:



Let the AC voltage applied be

$$V = V_m \sin\omega t$$

$$\therefore \quad V = \frac{q}{C}$$

Applying Kirchhoff's loop rule, we have

$$V_m \sin\omega t = \frac{q}{C} \Rightarrow q = CV_m \sin\omega t$$

Also, $$i = \frac{dq}{dt}$$

$$\Rightarrow \quad i = \frac{d}{dt}(Vm \sin\omega t)$$

$$i = \omega CV_m \cos\omega t \quad \ldots(i)$$

We know that $\cos\omega t = \sin\omega t + \pi/2$ ...(ii)

In the circuit, **(1)**

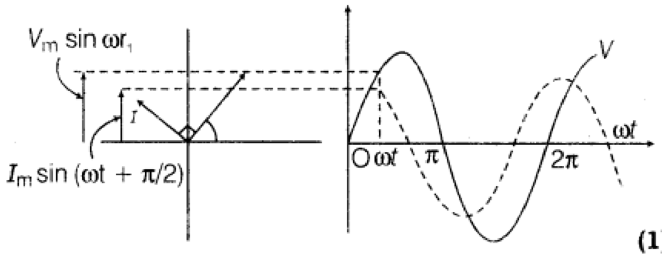
$$V_m = i_m X_C = i_m \frac{l}{\omega C}$$

$$\Rightarrow \quad i_m = V_m \omega C \quad \text{... (iii)}$$

Substituting the value of Eqs. (ii) and (iii) in Eq. (*i*) we get

$$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

The phase diagram which shows the current lead the voltage in phase by 90° is given below:



(1)

7. In a series L-C-R circuit, obtain the conditions under which (i) the impedance of circuit is minimum and (ii) wattless current flows in the circuit. [Foreign 2014]

Ans.

(*i*) The impedance of a series *L-C-R* circuit is given by $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

Z will be minimum when $\omega L = \frac{l}{\omega C}$, i.e.

When the circuit is under resonance Hence, or this condition Z will be minimum and equal to R. **(1)**

(*ii*) Average power dissipated through a series *L-C-R* circuit is given by

$$P_{av} = E_V I_V \cos\phi$$

where, E_V = rms value of alternating voltage

I_V = rms value of alternating current

ϕ = phase difference between current and voltage

For wattless current, the power dissipated through the circuit should be zero.

i.e. $\cos\phi = 0 \Rightarrow \cos\phi = \cos\frac{\pi}{2}$

$$\Rightarrow \quad \phi = \frac{\pi}{2}$$

Hence, the condition for wattless current is that the phase difference between the current and the circuit is purely inductive or purely capacitive. **(1)**

8. Calculate the quality factor of a series L-C-R circuit with $L = 2.0$ H, $C = 2\mu F$ and $R = 10\ \Omega$. Mention the significance of quality factor in L-C-R circuit. **[Foreign 2012]**

Ans.

Given,
$$L = 2.0 \text{ H}$$
$$C = 2\ \mu\text{F} = 2 \times 10^{-6} \text{ F}$$
$$R = 10\ \Omega$$

Now,
$$Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$
$$= \frac{1}{10}\sqrt{\frac{2}{2 \times 10^{-6}}} = \frac{1}{10 \times 10^{-3}}$$
$$= \frac{1}{10^{-2}} = 100$$ **(1)**

Quality factor is also defined as
$$Q = 2\pi f \times \frac{\text{Energy stored}}{\text{Power loss}}$$

9. An alternating voltage given by $V = 140 \sin 314t$ is connected across a pure resistor of $50\ \Omega$. Find

(*i*) the frequency of the source.

(*ii*) the rms current through the resistor. **[All India 2012]**

Ans.

Given, $V = 140 \sin 314t$, $R = 50\ \Omega$

Comparing it with $V = V_0 \sin \omega t$

(*i*) Here, $\omega = 314$ rad/s **(1/2)**

i.e. $2\pi\nu = 314$ $[\because \omega = 2\pi\nu]$

$$\Rightarrow \nu = \frac{314}{2\pi}$$

$$\nu = \frac{31400}{2 \times 314}$$

$$= 50 \text{ Hz}$$

Frequency of AC, $\nu = 50$ Hz **(1/2)**

(*ii*) As, $I_{rms} = \frac{V_{rms}}{R}$ and $V_{rms} = \frac{V_0}{\sqrt{2}}$ **(1/2)**

Here, $V_0 = 140$ V

$$\Rightarrow V_{rms} = \frac{140}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 70\sqrt{2} \text{ V}$$

$$\therefore I_{rms} = \frac{70\sqrt{2}}{R} = \frac{70\sqrt{2}}{50}$$

$$= 1.9 \text{ A or } 2 \text{ A}$$ **(1/2)**

10. An alternating voltage given by $V = 280 \sin 50\pi t$ is connected across a pure resistor of $40\ \Omega$. Find

(*i*) the frequency of the source.

(*ii*) the rms current through the resistor.

[All India 2012]

Ans.

Given, $V = 280 \sin 50\pi t$, $R = 40\ \Omega$

Comparing it with standard equation,

$$V = V_0 \sin \omega t$$

$$V_0 = 280 \text{ V}$$

$$\omega = 50\pi \text{ rad/s}$$

(*i*) As, $\omega = 2\pi\nu = 50\pi$ **(1/2)**

Frequency of AC, $\nu = \frac{50}{2\pi}\pi = 25$ Hz **(1/2)**

(*ii*) $$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{280}{\sqrt{2}} \text{ V}$$

$$\therefore I_{rms} = \frac{V_{rms}}{R} = \frac{280}{\sqrt{2} \times 40}$$

$$I_{rms} = \frac{280}{1.414 \times 40} = 4.95 \text{ A}$$

(1/2 × 2 = 1)

11. An alternating voltage given by $V = 70 \sin 100\,\pi t$ is connected across a pure resistor of 25 Ω. Find

(*i*) the frequency of the source.

(*ii*) the rms current through the resistor.

[All India 2012]

Ans.

Given, $V = 70 \sin 100\,\pi t$

On comparing with standard equation,

$$V = V_0 \sin \omega t \;\Rightarrow\; V_0 = 70\text{ V}$$

$$\omega = 100\,\pi \text{ rad/s}$$

(*i*) As, $\omega = 2\pi\nu = 100\,\pi$

Frequency of the source,

$$\nu = 50\text{ Hz}$$ **(1/2 × 2 = 1)**

(*ii*) $$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{70}{\sqrt{2}}$$

$$\therefore \quad I_{rms} = \frac{V_{rms}}{R}$$

$$= \frac{(70/\sqrt{2})}{25} = \frac{70}{1.414 \times 25}$$

$$= 1.98\text{ A}$$ **(1)**

12. (*i*) An alternating voltage $V = V_m \sin \omega t$ applied to a series *L-C-R* circuit derives a current given by $I = I_m \sin(\omega t + \phi)$. Deduce an expression for the average power dissipated over a cycle.

(*ii*) For circuit used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

[Foreign 2011; Delhi 2009]

Ans.

(i) Let at any instant, the current and voltage in an L-C-R series AC circuit is given by

$$I = I_0 \sin \omega t$$
$$V = V_0 \sin (\omega t + \phi)$$

The instantaneous power is given by

$$P = VI = V_0 \sin (\omega t + \phi) I_0 \sin \omega t$$
$$P = \frac{V_0 I_0}{2} [2 \sin \omega t \sin (\omega t + \phi)]$$
$$P = VI = \frac{V_0 I_0}{2} [\cos \phi - \cos (2\omega t + \phi)] \ldots \text{(i)}$$

$[\because 2 \sin A \sin B = \cos (A - B) - \cos (A + B)]$

Work done for a very small time interval dt is given by

$$dW = Pdt$$
$$dW = VIdt$$

$\therefore$ Total work done over T, a complete cycle is given by

$$W = \int_0^T VIdt$$ **(1/2)**

But, $P_{av} = \frac{W}{T} = \frac{\int_0^T VIdt}{T}$

$$\Rightarrow P_{av} = \frac{1}{T}\int_0^T VIdt$$
$$= \frac{1}{T}\int_0^T \frac{V_0 I_0}{2} [\cos \phi - \cos (2\omega t + \phi)]\, dt$$

or $P_{av} = \frac{V_0 I_0}{2T}\left[\int_0^T \cos \phi \, dt - \int_0^T \cos (2\omega t + \phi)\, dt\right]$

$$= \frac{V_0 I_0}{2T} [\cos \phi [t]_0^T - 0]$$

(By trigonometry)

or $P_{av} = \frac{V_0 I_0}{2T} \times \cos \phi \times T = \frac{V_0 I_0}{2} \times \cos \phi$

$$P_{av} = \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \cos \phi$$

$\Rightarrow P_{av} = V_{rms} \times I_{rms} \times \cos \phi$ **(1/2)**

This is required expression.

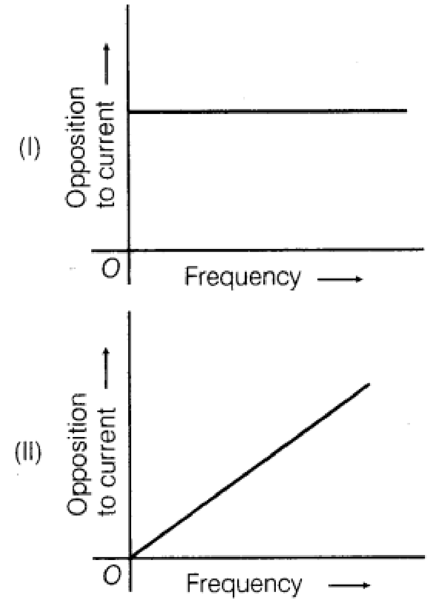
(ii) $\because$ Power factor, $\cos \phi = \frac{R}{Z}$

where, R = resistance and Z = impedance

Low power factor ($\cos \phi$) implies lower ohmic resistance and higher power loss as $P_{av} \propto \frac{1}{R}$ in power system (transmission line) **(1)**

13.(i) The graphs (I) and (II) represent the variation of the opposition offered by the circuit element to the flow of alternating current with frequency of the applied emf. Identify the circuit element corresponding to each graph.

(ii) Write the expression for the impedance offered by the series combination of the above two elements connected across the AC sources. Which will be ahead in phase in this circuit, voltage or current?



[All India 2011C]

Ans.

(*i*) From graph (I), it is clear that resistance (opposition to current) is not changing with frequency, i.e. resistance does not depend on frequency of applied source, so the circuit element here is pure resistance (R).

From graph (II), it is clear that resistance increases linearly with frequency, so the circuit element here is an inductor.

Inductive resistance, $X_L = 2\pi fL$

$\Rightarrow \quad X_L \propto f$ **(1)**

(*ii*) Impedance offered by the series combination of resistance R and inductor L.

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi fL)^2}$$

In L-R circuit, the applied voltage leads the current in phase by $\frac{\pi}{2}$. **(1)**

14. An AC source of voltage V = V_m sin wt is applied across a series L-C-R Draw the phasor diagram for this circuit when the

- capacitative impedance exceeds the inductive impedance.
- inductive impedance exceeds the capacitative impedance. [All India 2008C]

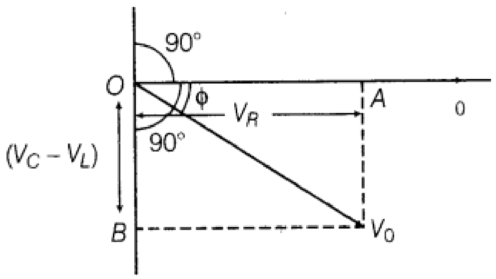
Ans.

(*i*) Capacitive impedance exceeds the inductive impedance.

i.e. $X_C > X_L$ or $V_C > V_L$

$$\left[\because X_C = \frac{V_C}{I} \text{ and } X_L = \frac{V_L}{I}\right]$$

Phase difference in an *L-C-R* circuit $\tan \phi = \frac{X_L - X_C}{R}$. Here, $X_C > X_L$, so $\tan \phi$ will be negative. Hence, voltage lags behind the current by a phase angle ϕ. In this case, AC circuit is called as **capacitance dominated circuit.** The phasor diagram in this case is shown as below:



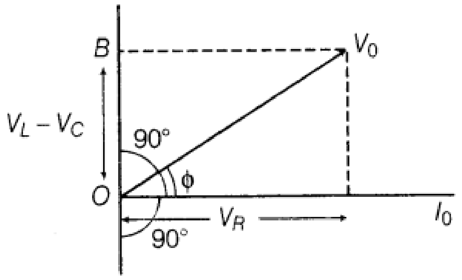
(1)

(*ii*) Inductive impedance exceeds the capacitive impedance, i.e. $X_L > X_C$ or $V_L > V_C$.

Phase difference, $\tan \phi = \frac{X_L - X_C}{R}$.

Here, $X_L > X_C$, so $\tan \phi$ is positive. Hence, voltage leads the current by a phase angle ϕ. In this case, AC circuit is called as **inductance dominated circuit.**

The phasor diagram in this case is shown as below:



(1)

15. An AC source of voltage V = V_m sin wt is applied across a

- series R-C circuit in which the capacitative impedance is a times the resistance in the circuit.
- series R – L circuit in which the inductive impedance is b times the resistance in the circuit.

Calculate the value of the power factor of the circuit in each case. [All India 2008C]

Ans.

Power factor is a ratio of the pure resistance to the total impedance of an AC circuit. So, to determine the power factor, first of all total impedance is to be calculated

$\because$ Power factor, $\cos\phi = \frac{R}{Z}$

where, ϕ = Phase difference between V and Z

R = Ohmic resistance

Z = Impedance.

(i) Given, $X_C = aR$

$\therefore$ Impedance, $Z = \sqrt{R^2 + X_C^2}$

$= \sqrt{R^2 + a^2R^2} = R\sqrt{1+a^2}$

$\therefore \cos\phi = \frac{R}{R\sqrt{1+a^2}} = \frac{1}{\sqrt{1+a^2}}$

Power factor $= \frac{1}{\sqrt{1+a^2}}$ **(1)**

(ii) Given, $X_L = bR$

$\therefore Z = \sqrt{R^2 + X_L^2}$

$= \sqrt{R^2 + b^2R^2}$

$Z = R\sqrt{1+b^2}$

$\therefore$ Power factor, $\cos\phi = \frac{R}{Z}$

$= \frac{R}{R\sqrt{1+b^2}} = \frac{1}{\sqrt{1+b^2}}$

$\therefore$ Power factor $= \frac{1}{\sqrt{1+b^2}}$ **(1)**

3 Marks Questions

16. A voltage V = V_0 sin wt is applied to a series L-C-R Derive the expression for the average power dissipated over a cycle.

Under what condition is (i) no power dissipated even though the current flows through the circuit (ii) maximum power dissipated in the circuit? [All India 2014]

Ans.

Let applied alternating voltage,

$$V = V_0 \sin \omega t \quad \text{... (i)}$$

We know that,

$$I = I_0 \sin(\omega t - \phi) \quad \text{... (ii)}$$

Power, $P = VI$ **(1/2)**

Substituting the value of V and I form Eq. (i) and Eq. (ii), we get

$P = V_0 \sin \omega t \,.\, I_0 \sin(\omega t - \phi)$

So, the instantaneous power is given by

$\because \sin(\omega t - \phi) = \sin \omega t \cos \phi - \cos \omega t \sin \phi$

(*i*) If $\phi = 90°$, then no power is dissipated even though the current flows through the circuit.

$$P_{av} = 0$$

$$\because \quad \tan \phi = \left(\frac{X_L - X_C}{R}\right)$$

$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R} = \infty \quad (\because \tan 90° = \infty)$$

(1/2)

(*ii*) If $\phi = 0°$, then maximum power is dissipated in the circuit.

$$P_{av} =$$

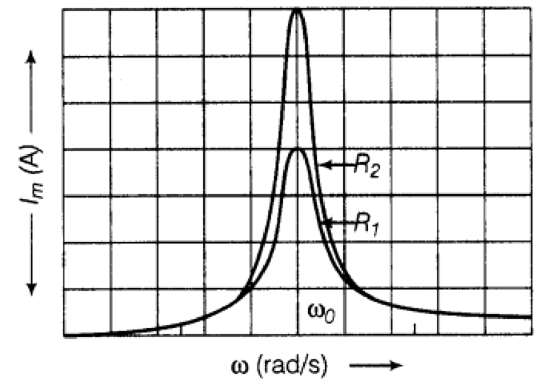
$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R} = 0 \quad (\because \tan 0° = 0)$$

$\Rightarrow X_L = X_C$ (Resonance) **(1/2)**

17. If a series L-C-R circuit connected to an AC source of variable frequency and voltage $V = V_m \sin \omega t$, draw a plot showing the variation of current I with angular frequency ω for two different values of resistances R_1 and R_2 $(R_1 > R_2)$. Write the condition under which the phenomenon of resonance occurs. For which value of the resistance out of the two curves, a sharper resonance is produced? Define Q-factor of the circuit and give its significance. **[Delhi 2013]**

Ans.

Figure shows the variation of i_m with ω in a L-C-R series circuit for two values of resistance R_1 and R_2 ($R_1 > R_2$).



(1)

The condition for resonance in the L-C-R circuit is $X_L = X_C$

$$\omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 = \frac{1}{LC} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

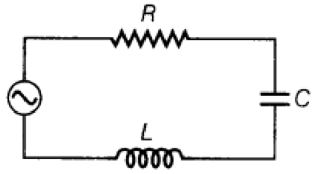
We see that the current amplitude is maximum at the resonant frequency ω_0. Since, $i_m = V_m/R$ at resonance, the current amplitude for case R_2 is sharper to that for case R_1. **(1)**

Quality factor or simply the Q-factor of a resonant L-C-R circuit is defined as the ratio of voltage drop across the capacitor (or inductor) to that of applied voltage.

It is given by $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

The Q-factor determines the sharpness of the resonance curve and if the resonance is less sharp, not only is the maximum current less, the circuit is close to the resonance for a larger range $\Delta\omega$ of frequencies and the tuning of the circuit will not be good. So, less sharp the resonance, less is the selectivity of the circuit while higher is the Q, sharper is the resonance curve and lesser will be the loss in energy of the circuit. **(1)**

18. The figure shows a series L-C-R circuit with $L =$ 10.0 H, $C = 40\ \mu F$, $R = 60\ \Omega$ connected to a variable frequency 240 V source. Calculate



(*i*) the angular frequency of the source which derives the circuit at resonance.

(*ii*) the current at the resonating frequency.

(*iii*) the rms potential drop across the inductor at resonance. **[Delhi 2012]**

Ans.

Given, $L = 10$ H, $C = 40\ \mu F$,
$R = 60\ \Omega$, $V_{rms} = 240$ V

(*i*) Resonating angular frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 40 \times 10^{-6}}}$$

$$\omega_0 = \frac{1}{20 \times 10^{-3}} = 50 = 50 \text{ rad/s}$$ **(1)**

(*ii*) Current at resonating frequency,

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} \quad (\because \text{At resonance } Z = R)$$

$$= \frac{240}{60} = 4 \text{ A} = 4 \text{ A}$$ **(1)**

(*iii*) $\because$ Inductive reactance, $X_L = \omega L$

At resonance,

$$X_L = \omega_0 L = 50 \times 10 = 500\ \Omega$$

Potential drop across to inductor,

$$V_{rms} = I_{rms} \times X_L = 4 \times 500$$

$$= 2000 \text{ V}$$ **(1)**

19.A series L-C-R circuit is connected to an AC source. Using the phasor diagram, derive the expression for the impedance of the circuit. Plot a graph to show the variation of current with frequency of the source, explaining the nature of its variation. [All India 2012]

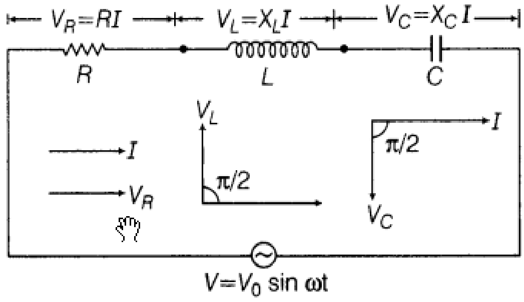
Ans.

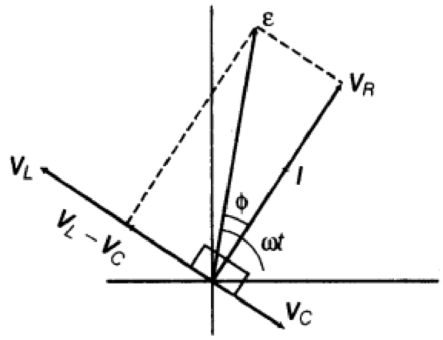
Assuming $X_L > X_C$,

$\Rightarrow$ $V_L > V_C$

$\because$ Net voltage, $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

where, V_L, V_C and V_R are PD across L, C and R respectively.





(2)

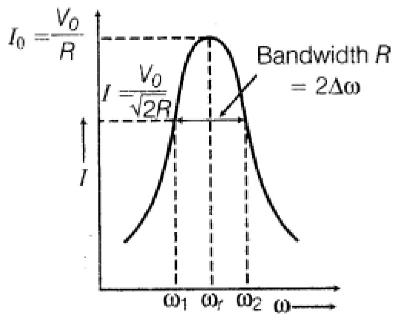
But, $V_R = IR, V_L = IX_L, V_C = IX_C$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$\frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

Impedance of L-C-R circuit,

$$Z = \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$ **(1)**



20. A series L-C-R circuit is connected to a 220 V variable frequency AC supply. If $L = 20$ mH, $C = (800/\pi^2)\,\mu F$ and $R = 110\,\Omega$.

(*i*) Find the frequency of the source for which average power absorbed by the circuit is maximum.

(*ii*) Calculate the value of maximum current amplitude. **[Delhi 2010C]**

Ans.

Given, $V_{rms} = 220$ V, $L = 20$ mH $= 2 \times 10^{-2}$ H,

$$R = 110\ \Omega,$$

$$C = \frac{800}{\pi^2}\ \mu F = \frac{800}{\pi^2} \times 10^{-6}\ F$$

(i) Average power observed by *L-C-R* series AC circuit is maximum when circuit is in resonance.

∴ Resonant frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow = \frac{1}{2\pi\sqrt{2 \times 10^{-2} \times \frac{800}{\pi^2} \times 10^{-6}}}$$

$$\nu_0 = \frac{1000}{2 \times 4} = 125\ s^{-1}$$

$$\nu_0 = 125\ s^{-1}$$ $\left(1\frac{1}{2}\right)$

(ii) As, $I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{110} = 2$ A

$$Z = R = 110\ \Omega$$

$$I_{rms} = 2\ A$$

∴ Maximum current amplitude,

$$I_0 = I_{rms}\sqrt{2} = 2\sqrt{2}\ A$$ $\left(1\frac{1}{2}\right)$

21. An AC voltage V = V_0 sin wt is applied across a pure inductor Obtain an expression for the current / in the circuit and hence obtain the

- inductive reactance of the circuit and
- the phase of the current flowing with respect to the applied voltage. [All India 2010C]

Ans.

> Due to change in flux, the emf is induced in the coil. The rate of change of flux will give the value of emf.

Let an alternating voltage, $V = V_0 \sin \omega t$ is applied across pure inductor of inductance L. The magnitude of induced emf is given by

$$e = L\frac{dI}{dt}$$ **(1)**

For the circuit,
Magnitude of induced emf = Applied voltage

i.e. $$L\frac{dI}{dt} = V_0 \sin \omega t$$

or $$dI = \frac{V_0}{L} \sin \omega t\ dt$$

On integrating both sides, we get

$$I = \frac{V_0}{L}\int \sin \omega t \, dt = \frac{V_0}{L}\left(\frac{-\cos \omega t}{\omega}\right) \quad \textbf{(1)}$$

or $$I = -\frac{V_0}{\omega L}\cos \omega t = -\frac{V_0}{\omega L}\sin\left(\frac{\pi}{2} - \omega t\right)$$

$\therefore$ $$I = \frac{V_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{...(i)}$$

where, $X_L = \omega L$ = inducting reactance

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{...(ii)}$$

where, I_0 = peak value of AC

But, $V = V_0 \sin \omega t$ ...(iii)

From Eqs. (ii) and (iii), it is clear that AC lags behind the voltage by phase $\frac{\pi}{2}$. **(1)**

22. An AC voltage, $V = V_0 \sin \omega t$ is applied across a pure capacitor, C. Obtain an expression for the current I in the circuit and hence obtain the

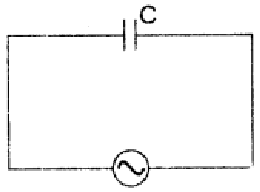
(*i*) capacitive reactance of the circuit and

(*ii*) the phase of the current flowing with respect to the applied voltage.

[All India 2010C]

Ans.

(*i*) Let alternating voltage, $V = V_0 \sin \omega t$ is applied across a capacitor C. At any instant, the potential difference across the capacitor is equal to applied voltage. **(1)**



$V = V_0 \sin \omega t$ ...(i)

$\therefore V =$ Potential difference across the capacitor $= \frac{q}{C}$

$$\Rightarrow \quad q = CV$$

or $\quad q = CV_0 \sin \omega t$

$$\therefore \quad \frac{dq}{dt} = \omega C V_0 \cos \omega t$$

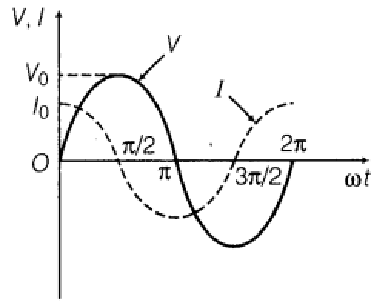
or $$I = \frac{V_0}{\left(\frac{1}{\omega C}\right)} \cos \omega t$$

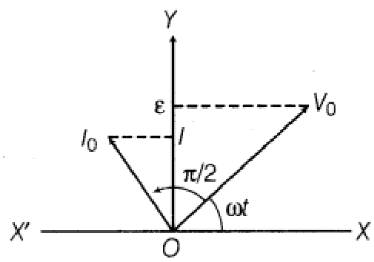
or $$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{...(ii)}$$

where, $I_0 = \dfrac{V_0}{\left(\frac{1}{\omega C}\right)} = \dfrac{V_0}{X_C}$

Capacitive reactance, $X_C = \dfrac{1}{\omega C}$ **(1)**

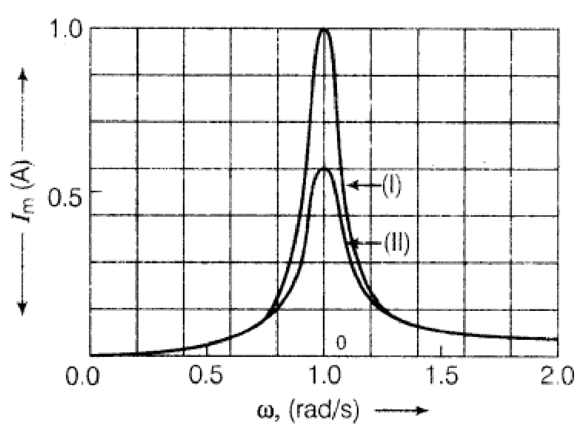
(ii) From Eqs. (i) and (ii), current leads the voltage by phase $\dfrac{\pi}{2}$.





(1)

23. The graphs shown here depict the variation of current I_{rms} with angular frequency ω for two different series *L-C-R* circuits.



Observe the graphs carefully.

(i) State the relation between L and C values of the two circuits when the current in the two circuits is maximum.

(ii) Indicate the circuit for which

- power factor is higher
- quality factor Q is larger.

Give the reasons for each case. [Delhi 2009C]

Ans.

(*i*) Since, resonant angular frequency ω_0 is same for both the graphs.

i.e. $(\omega_0)_1 = (\omega_0)_2$ $\left(\because \omega_0 = \frac{1}{\sqrt{LC}}\right)$

$\Rightarrow \frac{1}{\sqrt{L_1C_1}} = \frac{1}{\sqrt{L_2C_2}}$

$\Rightarrow L_1C_1 = L_2C_2$

or $\frac{L_1}{L_2} = \frac{C_2}{C_1}$ **(1)**

(*ii*) (a) In graph (I), current I_{m_1} is greater than graph (II) I_{m_2}.

i.e. $I_{m_1} > I_{m_2}$

$\Rightarrow R_1 < R_2$

Power factor of circuit represented by graph (I) has power factor as $\cos\phi \propto R$. **(1)**

(*b*) $\because Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ [For two graphs]

i.e. $Q \propto \frac{1}{R}$

As, $R_1 < R_2$

$\therefore Q_1 > Q_2$

Quality factor of graph (I) is higher than that of graph (II). **(1)**

24. An inductor 200 mH, capacitor 500 µF, resistor 10 Ω are connected in series with a 100 V variable frequency AC source. Calculate the

(*i*) frequency at which the power factor of the circuit is unity.

(*ii*) current amplitude at this frequency.

(*iii*) Q-factor. **[Delhi 2008; Foreign 2008]**

Ans.

Given, $L = 200 \text{ mH} = 2 \times 10^{-1}\text{H}$

$C = 500\,\mu\text{F} = 5 \times 10^{-4}\text{F}$

(*i*) When power factor $\cos\phi = 1$

Then, $\phi = 0$

$\Rightarrow$ *L-C-R* circuit is in resonance.

$\therefore$ Resonating frequency, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{2 \times 10^{-1} \times 5 \times 10^{-4}}}$$

$$\omega_0 = 100 \text{ s}^{-1}$$ **(1)**

(*ii*) We know that, $I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R}$

[$\because$ at resonance condition, $Z = R$]

$\therefore \quad I_{rms} = \frac{100}{10} = 10 \text{ A}$

Amplitude of maximum current, $I_0 = I_{rms}\sqrt{2}$

$= 10\sqrt{2} \text{ A}$ **(1)**

(*iii*) As, Q-factor $= \frac{1}{R}\sqrt{\frac{L}{C}}$

$$= \frac{1}{10}\sqrt{\frac{2 \times 10^{-1}}{5 \times 10^{-4}}} = \frac{1}{10}\sqrt{0.4 \times 10^3}$$

$$= \frac{20}{10} = 2$$ **(1)**

$\therefore$ Q- factor $= 2$

25. An inductor and a bulb are connected in series to an AC source of 12 V, 50 Hz. 2 A current flows in the circuit and the phase angle between voltage and current is $\frac{\pi}{4}$ rad. Calculate the impedance and inductance of the circuit. **[Foreign 2008]**

Ans.

To find out the inductance firstly, we have to calculate (Z) impedance and resistance. Here, resistance can be calculated by the given value of ϕ and calculated value of Z means formula of power factor can be applied.

Given, $V_{rms} = 12$ V, $f = 50$ Hz, $\phi = \frac{\pi}{4}$ rad

$I_{rms} = 2$ A

$\therefore$ Impedance, $Z = \frac{V_{rms}}{I_{rms}} = \frac{12\text{ V}}{2\text{ A}} = 6\ \Omega$ **(1)**

Also, power factor, $\cos\phi = \frac{R}{Z}$

$$\Rightarrow \quad R = Z\cos\phi$$

$$= 6 \times \cos\frac{\pi}{4} = 6 \times \frac{1}{\sqrt{2}}$$

$$R = 3\sqrt{2}\ \Omega$$ **(1/2)**

In C-R circuit, $R^2 + X_L^2 = Z^2$

$$X_L^2 = Z^2 - R^2 = (6)^2 - (3\sqrt{2})^2$$

$$\Rightarrow \quad X_L = 3\sqrt{2}\ \Omega$$ **(1/2)**

But, $X_L = 2\pi f L = 3\sqrt{2}$

Inductance, $L = \frac{3\sqrt{2}}{2\pi f} = \frac{3\sqrt{2}}{2 \times 3.14 \times 50}$

$= 13.5$ mH **(1)**

26. A coil of $L = 0.5\pi\ \Omega$ is connected to a 200 V, 50 s^{-1} AC source. Calculate the following

(*i*) Maximum current in the coil.

(*ii*) Time lag between voltage maximum and current maximum. **[Foreign 2008]**

Ans.

(*i*) Given, $V_{rms} = 200$ V,

$f = 50\ s^{-1}, L = 0.5\pi\ \Omega$

Inductive reactance, $X_L = \omega L = 2\pi f L$

$= 2\pi \times 0.5\pi \times 50 = 492.98\ \Omega$

$\approx 493\ \Omega$

Maximum current,

$$I_0 = \frac{V_0}{X_L} = \frac{V_{rms}\sqrt{2}}{X_L} = \frac{200\sqrt{2}}{493}$$

$= 0.4 \times 1.414 = 0.57$ A **(1)**

(*ii*) Phase difference $= \frac{2\pi}{T} \times$ Time difference

$\because$ For pure inductor, phase difference

$\phi = \frac{\pi}{2}, \quad T = \frac{1}{f} = \frac{1}{50}$ s **(1)**

$\therefore$ Time difference $= \frac{T}{2\pi} \times$ Phase difference

$$= \frac{\left(\frac{1}{50}\right)}{2\pi} \times \frac{\pi}{2}$$

i.e. Time lag between V and I is $\frac{1}{200}$ s **(1)**

27. A coil of inductance, 0.5 H and resistance, 100 Ω is connected to a 200 V, 50 s^{-1} AC source. Calculate

(*i*) the maximum current in the coil.

(*ii*) time lag between voltage maximum and current maximum. **[Foreign 2008]**

Ans. Refer Q.no 26

28. When a given coil is connected to a 200 V DC source, 2 A current flows and when the same coil is connected to 200 V, 50 Hz AC source, only 1 A current flows in the circuit.

- Explain why the current decreases in the latter case?
- Calculate the self-inductance of the coil used. [Foreign 2008]

Ans.

In DC circuit, an inductor offers no resistance, so the impedance of the circuit is equal to the resistance in the circuit (Z = R).

(*i*) In DC circuit, $X_L = 2\pi fL = 0$ $(\because f = 0)$

$\therefore$ But a coil have impedance,

$$Z = \sqrt{R^2 + X_L^2}$$

In DC circuit, $Z = R$ (as $X_L = 0$)

But in AC, $Z = \sqrt{R^2 + X_L^2} > R$

(X_L have certain value in AC)

i.e. $I \propto \frac{1}{Z}$

So, in DC more current flows than AC.

$\left(1\frac{1}{2}\right)$

(*ii*) In DC circuit,

$$R = \frac{V}{I} = \frac{200}{2} = 100\ \Omega$$

In AC circuit, $Z = \frac{V}{I} = \frac{200}{1} = 200\ \Omega$ **(1)**

$$\Rightarrow \quad R^2 + X_L^2 = Z^2$$

$$X_L^2 = Z^2 - R^2 = (200)^2 - (100)^2$$

$$X_L = 100\sqrt{3}\ \Omega$$

$$\Rightarrow \quad 2\pi fL = 100\sqrt{3}\ \Omega$$

$\therefore$ Inductance, $L = \frac{100\sqrt{3}}{2\pi f}$

Given, $f = 50\text{Hz}$

$$= \frac{100\sqrt{3}}{2 \times 3.14 \times 50}$$

$f = 0.55$ H **(1/2)**

29. A 100 μF capacitor is in series with a 40 Ω resistor and is connected to a 100 V, 50 Hz AC source. Calculate the following

(*i*) Maximum current in the circuit.

(*ii*) Time lag between current maximum and voltage maximum. **[Foreign 2008]**

Ans.

(i) To calculate the maximum current in circuit, firstly find the impedance (Z) and rms value of current.

(ii) It can be calculated with the help of formula of phase difference.

(i) Given, $V_{rms} = 100$ V, $C = 100\ \mu F$,

$$C = 100 \times 10^{-6}\ F$$

$$R = 40\ \Omega \quad \textbf{(1)}$$

Total impedance of the circuit,

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(R)^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$Z = \sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}$$

$$= \sqrt{(40)^2 + \left(\frac{1}{2 \times 3.14 \times 50 \times 100 \times 10^{-6}}\right)^2}$$

$$= 51.12\ \Omega \quad \textbf{(1)}$$

Current in the circuit, $I_{rms} = \frac{V_{rms}}{Z}$

$$= \frac{100}{51.12}$$

$$= 1.95\ A$$

Maximum current, $I_0 = I_{rms}\sqrt{2}$

$$= 1.95\sqrt{2}$$

$$= 2.76\ A$$

(ii) For pure capacitor, phase difference, $\phi = \frac{\pi}{2}$

Time difference $= \frac{T}{2\pi} \times$ Phase difference

$$= \frac{\frac{1}{50}}{2\pi} \times \frac{\pi}{2} = \frac{1}{200}\ s \quad \textbf{(1)}$$

$\therefore$ Time lag between current maximum and voltage maximum $= \frac{1}{200}$ s.

30. An AC source of voltage $V = V_m \sin \omega t$ is connected one by one to three circuit elements X, Y and Z. It is observed that the current flowing in them

(i) is in phase with applied voltage for element X.

(ii) leads the voltage in phase by $\pi/2$ for element Y.

(iii) lags the applied voltage in phase by $\pi/2$ for element Z.

Identify the three circuit elements.

Find an expression for the

(a) current flowing in the circuit.

(b) net impedance of the circuit when the same AC source is connected across a series combination of the elements, X, Y and Z.

If the frequency of the applied voltage is varied, then set up the condition of the frequency when the current amplitude in the circuit is maximum. Write the expression for this current amplitude. **[HOTS; Delhi 2008C]**

Ans.

> We will identify the element according to the phase lead and lag with voltage or current.

(i) In element X, current is flowing in phase with applied voltage, so element X is a resistance.

(ii) In element Y, current leads the applied voltage in phase by $\frac{\pi}{2}$, so element Y is a capacitor.

(iii) In element Z, current lags the applied voltage in phase by $\frac{\pi}{2}$, so element Z is an inductor. **(1)**

(a) Current flowing in the circuit,

$$I = I_0 \sin(\omega t + \phi), \text{ if } X_C > X_L$$
$$I = I_0 \sin(\omega t - \phi), \text{ if } X_L > X_C$$

where, $\tan\phi = \frac{X_L \sim X_C}{R}$ **(1/2)**

(b) Impedance of the circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where, $X_L = \omega L$, $X_C = \frac{1}{\omega C}$ **(1/2)**

Current amplitude will be at resonance.

At resonance, $X_L = X_C$

$\therefore \quad Z = R$

Thus, maximum current of amplitude,

$$I_0 = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R}$$ **(1)**

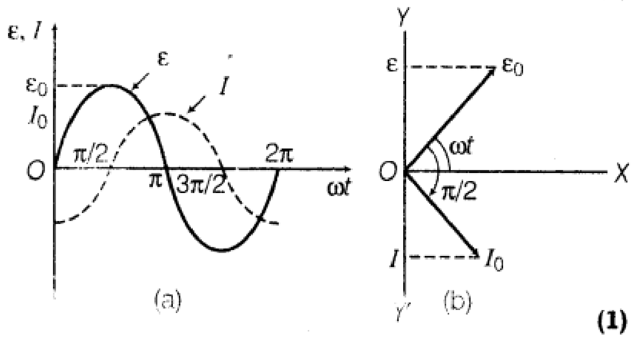
31. What does the term 'phasors' in AC circuit analysis mean? An AC source of voltage, V = V_m sin wt is applied across a pure inductor of inductance L. Obtain an expression for the current I flowing in the circuit. Also draw the

- phasor diagram.
- graphs of V and I versus wt for this circuit. [Delhi 2008C]

Ans.

Phasor A phasor is a vector which rotates about the origin with an angular speed ω in anti-clockwise direction that representing it as a sinusoidally varying quantity (like alternating emf and alternating current). Its length represents the amplitude of the quantity and projection on a fixed axis given the instantaneous value of represented quantity. **(1)**

For explanation of the current Refer to ans. 19. **(1)**



(1)

32. A series *L-C-R* circuit is made by taking $R = 100\ \Omega$, $L = \frac{2}{\pi}$ H, $C = \frac{100}{\pi}\ \mu$F. This series combination is connected across an AC source of 220 V, 50 Hz. Calculate

(*i*) the impedance of the circuit and

(*ii*) the peak value of the current flowing in the circuit.

(*iii*) the power factor of this circuit and compare this value with the one at its resonant frequency. **[All India 2008C]**

Ans.

Firstly, to find out the impedance, we have to find inductive reactance (X_L) and capacitive reactance (X_C), then can find peak value of current and power factor.

(i) Given, $R = 100\ \Omega$, $L = \frac{2}{\pi}$ H, $C = \frac{100}{\pi}\ \mu$H,

$V = 200$ V, $f = 50$ Hz.

$\because$ Inductive reactance of AC circuit,

$$X_L = \omega L = 2\pi f L = 2\pi \times 50 \times \frac{2}{\pi} = 200\ \Omega$$

Capacitive reactance of AC circuit,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi \times 50 \times \left(\frac{100}{\pi}\right) \times 10^{-6}} = 100\ \Omega$$

Impedance of *L-C-R* series AC circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{(100)^{Z2} + (200 - 100)^2}$$

$$= 100\sqrt{2}\ \Omega$$ **(1)**

(ii) Peak value of current flowing through the circuit, $I_0 = \frac{V_0}{Z} = \frac{220\sqrt{2}}{100\sqrt{2}}$

$(\because V_{rms} = 200\ \text{V}, V_0 = \sqrt{2}V_{rms} = 200\sqrt{2}\ \text{V})$

$$I_0 = 2.2\ \text{A}$$ **(1)**

(iii) Power factor, $\cos\phi = \frac{R}{Z}$

$$\cos\phi = \frac{100}{100\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$\therefore$ Power factor at resonance, $\cos\phi = 1$.

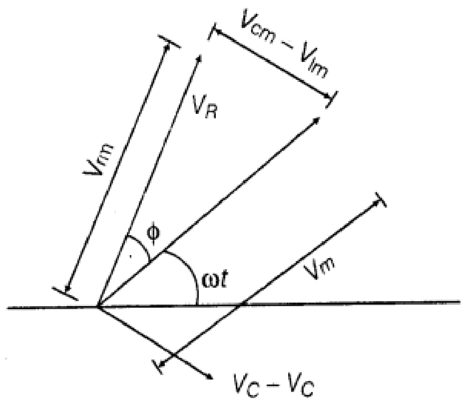
For comparison, $\frac{(\cos\phi)_{\text{given circuit}}}{(\cos\phi)_{\text{resonant circuit}}} = \frac{1}{\sqrt{2}}$ **(1)**

5 Marks Questions

33. (i) A series L-C-R circuit is connected to an AC source of variable frequency. Draw a suitable phasor diagram to deduce the expressions for the amplitude of the current and phase angle.

(ii) Obtain the condition at resonance. Draw a plot showing the variation of current with the frequency of AC source for two resistances R_1 and R2 (R_1 > R2). Hence, define the quality factor Q and write its role in the tuning of the circuit. [Delhi 2014C]

Ans.



From the phasor diagram.

$$V = V_L \quad V_R + V_C$$

Magnitude of net voltage,

$$V_m = \sqrt{(V_{rm})^2 + (V \quad - V \quad)^2}$$

$$V_m = I_m \sqrt{[R^2{}_m + (X_C - X_L)^2]}$$

From the figure,

$$\tan\phi = \frac{V_{cm} - V_{lm}}{V_{rm}}$$ **(2)**

$$= \frac{I_m (X_C - X_L)}{I_{mr}}$$

$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

(*b*) Refer to ans. 19. **(3)**

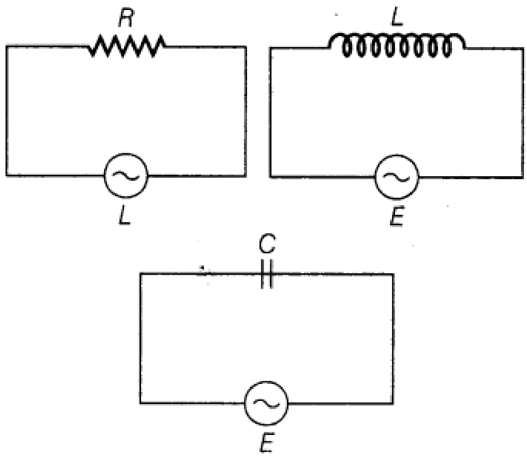
34. Derive an expression for the impedance of a series L-C-Rcircuit connected to an AC supply of variable frequency.

Plot a graph showing variation of current with the frequency of the applied voltage. Explain briefly how the phenomenon of resonance in the circuit can be used in the tuning mechanism of a radio or a TV set? [Delhi 2011]

Ans.Refer to ans.19 and for resonance condition refer to ans. 30(b)

35. (i) What do you understand by sharpness of resonance in a series L-C-R circuit? Derive an expression for O-factor of the circuit.

(ii) Three electrical circuits having AC sources of variable frequency are shown in the figures. Initially, the current flowing in each of these is same. If the frequency of the applied AC source is increased, how will the current flowing in these circuits be affected? Give the reason for your answer.

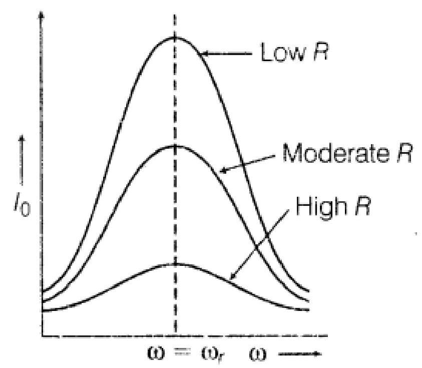


[HOTS; Delhi 2011C]

Ans.

On increasing the frequency of AC, there will be no effect on resistance (as it does not depend on frequencies). Inductive reactance will increase (as $X_L = 2\pi fL$ or $X_L \propto f$), capacitive reactance will decrease (as $X_C = \frac{1}{2\pi fC}$ or $X_C \propto \frac{1}{f}$). If resistance or reactance of a circuit increases/decreases, the current for the circuit decreases/increases.

(*i*)



i.e. Quality factor, $Q = \frac{\text{voltage across } L \text{ or } C}{\text{applied voltage}}$

$$Q = \frac{(\omega_r L)I}{RI}$$

[$\because$ applied voltage = voltage across R]

or $$Q = \frac{\omega_r L}{R}$$

or $$Q = \frac{(1/\omega_r C)I}{RI} = \frac{1}{RC\omega_r}$$

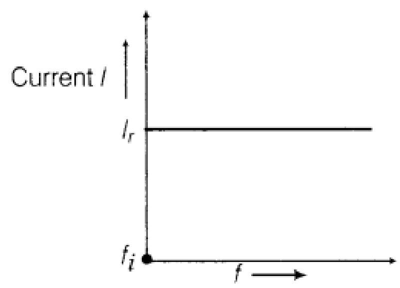
$\therefore$ $$Q = \frac{L}{RC \cdot \frac{1}{\sqrt{LC}}} \quad \left[\text{using } \omega_r = \frac{1}{\sqrt{LC}}\right]$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

or $$Q = \frac{1\sqrt{LC}}{RC} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \left[\text{using } \omega_r = \frac{1}{\sqrt{LC}}\right]$$

Thus, $$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

(a) **Circuit containing resistance *R* only** There will not be any effect in the current on changing the frequency of AC source. **(1)**



where, f_i = initial frequency of AC source.

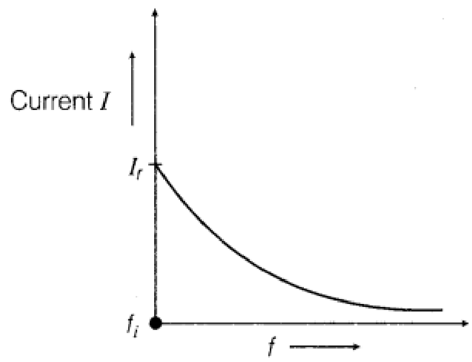
There is no effect on current with the increase in frequency.

(*b*) **AC circuit containing inductance only** With the increase of frequency of AC source inductive reactance increase as

$$I = \frac{V_{rms}}{X_L} = \frac{V_{rms}}{2\pi f L}$$

For given circuit,

$$I \propto \frac{1}{f}$$



(1)

Current decreases with the increase of frequency.

(*c*) **AC circuit containing capacitor only**

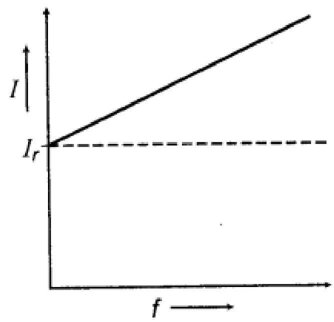
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Current, $I = \frac{V_{rms}}{X_C} = \frac{V_{rms}}{\left(\frac{1}{2\pi f C}\right)}$

$$I = 2\pi f C V_{rms}$$

For given circuit, $I \propto f$

Current increases with the increase of frequency.



(1)

36. A series L-C-Rcircuit is connected to an AC source having voltage V = V_m sin wt Derive the expression for the instantaneous current I and its phase relationship to the applied voltage. Obtain the condition for resonance to occur. Define power factor. State the conditions under which it is

- maximum and
- minimum. [All India 2010]

Ans.

Phase difference between voltage and current,

$$\tan\phi = \frac{X_L - X_C}{R} \quad \text{...(i)}$$

and $$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{(X_L - X_C)^2 + R^2}}$$

∴ Expression of AC,

$$I = I_0 \sin(\omega t - \phi) \quad \textbf{(1)}$$

Conditions for resonance

(*i*) Inductive reactance must be equal to capacitive reactance

i.e. $X_L = X_C$

(*ii*) As, $X_L = X_C$

$$\Rightarrow \quad \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

where, ω_0 = resonant angular frequency.

(1)

(*iii*) Impedance becomes minimum and equal to ohmic resistance

i.e. $Z = Z_{minimum} = R$

(*iv*) AC becomes maximum,

$$\therefore \quad I_{max} = \frac{V_{max}}{Z_{min}} = \frac{V_{max}}{R} \quad \textbf{(1)}$$

(*v*) Voltage and current arrives in same phase.

Power factor In an AC circuit, the ratio of true power consumption and virtual power consumption is termed as power factor.

i.e. $$\cos\phi = \frac{P_{av}}{V_{rms} I_{rms}} = \frac{\text{True power}}{\text{Apparent power}}$$

Also, $$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} \quad \textbf{(1)}$$

The power factor is maximum.

i.e. $\cos\phi = +1$, in *L-C-R* series AC circuit when circuit is in resonance. The power factor is minimum when phase angle between *V* and *I* is 90°, i.e. either pure inductive circuit or pure capacitive AC circuit. **(1)**

37. A resistor of 400 Ω, an inductor of $\frac{5}{\pi}$ H and a capacitor of $\frac{50}{\pi}$ μF are connected in series across a source of alternating voltage of 140 sin 100 πt V. Find the voltage (rms) across the resistor, the inductor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

(Given, $\sqrt{2} = 1.414$). **[Foreign 2010]**

Ans.

> To find the voltage across each circuit element, following steps to be followed are:
>
> (*i*) To calculate the maximum current in circuit, firstly find the (Z) impedance and rms value of current.
>
> (*ii*) It can be calculated with the help of formula of phase difference.

As, applied voltage

$$V = 140 \sin 100\, \pi t$$

$$C = \frac{50}{\pi}\,\mu F,\ C = \frac{5}{\pi}\,H,\ R = 400\,\Omega$$

$$= \frac{50}{4} \times 10^{-6}\,F$$

Comparing it with $V = V_0 \sin \omega t$,

$$V_0 = 140\ V,\ \omega = 100\,\pi$$

Inductive reactance, $X_L = \omega L$

$$X_L = 100\pi \times \frac{5}{\pi} = 500\,\Omega$$

Capacitive reactance, $X_C = \frac{1}{\omega C}$

$$X_C = \frac{1}{100\pi \times \frac{50}{\pi} \times 10^{-6}} = 200\,\Omega$$

Impedance of the AC circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(400)^2 + (500 - 200)^2}$$

$$Z = \sqrt{1600 + 900} = 500\,\Omega \quad \textbf{(2)}$$

Maximum current in the circuit,

$$I_0 = \frac{V_0}{Z} = \frac{140}{500}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{140}{500 \times \sqrt{2}} = 0.2\ A \quad \textbf{(1)}$$

V_{rms} across resistor R, $V_R = I_{rms} R$

$= 0.2 \times 400 = 80$ V

V_{rms} across inductor, $V_L = I_{rms} X_L$

$= 0.2 \times 500 = 100$ V

V_{rms} across capacitor, $V_C = I_{rms} \times X_C$

$= 0.2 \times 200 = 40$ V **(2)**

Here, $V = V_R + V_L + V_C$

Because V_L and V_R are not in same phase,

$$\therefore \quad V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

38. (*i*) Derive an expression for the average power consumed in a series *L-C-R* circuit connected to AC source for which the phase difference between the voltage and the current in the circuit is ϕ.

(*ii*) Define the quality factor in an AC circuit. Why should the quality factor have high value in receiving circuits? Name the factors on which it depends. **[Delhi 2009]**

Ans. (i) Refer to ans 12(i)

(ii) Refer to ans. 35 (i)

Quality factor depends on ohmic resistance of L-C-R Series.Ac and Inductance and Capacitance of L-C-R Series AC Circuit.

39. An AC source of emf, $E = E_0 \sin \omega t$ is connected across a series combination of an inductor L, a capacitor C and a resistor, R. Obtain an expression for the equivalent impedance Z of the circuit and hence, find the value of ω for the AC source for which $Z = R$.

Show that the phase angle ϕ (between the current flowing in this circuit and the voltage applied to it) can be obtained through the relation.

$$\phi = \tan^{-1}\left(\frac{\text{Reactive impedance}}{\text{Resistance}}\right)$$

[All India 2009]

Ans.

For impedance Refer to ans. 19. **(1)**

Also, from phasor diagram

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{\text{Reactive impedance}}{\text{Resistance}}$$

$$\Rightarrow \quad \phi = \tan^{-1}\left[\frac{\text{Reactive impedance}}{\text{Resistance}}\right]$$ **(1)**

The impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where, $Z = R$

$$\Rightarrow \quad R = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow \quad (X_L - X_C)^2 = 0$$

$$X_L = X_C$$

$$\Rightarrow \quad \omega_0 L = \frac{1}{\omega_0 C}$$

$$\Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}}$$ **(3)**

where, ω_0 is resonant L-C-R series AC circuit.

40. An AC source generating a voltage,

(*i*) $V = V_m \sin \omega t$ is connected to a capacitor of capacitance, C. Find the expression for the current I, flowing through it. Plot a graph of V and I *versus* ωt to show that the current is $\pi/2$ ahead to a voltage.

(*ii*) A resistor of 200 Ω and a capacitor of 15.0 μF are connected in series to a 200 V, 50 Hz AC source. Calculate the current in the circuit and the rms voltage across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox. **[All India 2008]**

Ans.

(i) Refer to ans. 22. **(2)**

(ii) Given, $R = 200\ \Omega$, $C = 40\ \mu F$, $V_{rms} = 220\ V$, $\omega = 300$ Hz

Inductive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{300 \times 40 \times 10^{-6}}$$ **(1/2)**

$$= 83.33\ \Omega$$

Impedance, $Z = \sqrt{R^2 + X_C^2}$

$$= \sqrt{(200)^2 + (83.33)^2}$$

$$= 216.6\ \Omega \quad \text{(nearly)}$$

$$\therefore \quad I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{216.6} = 1.02\ A$$ **(1/2)**

Rms voltage across R,

$$V_R = IR = 1.02 \times 200 = 204\ V$$ **(1)**

Across capacitor,

$$V_C = I X_C = 1.02 \times 83.33$$

$$V_C = 85\ V$$

$$V \neq V_R + V_C$$

Because V_C and V_R are not in same phase **(1/2)**

Hence, V_R leads V_C by phase $\frac{\pi}{2}$.

$$\therefore \quad V = \sqrt{V_R^2 + V_C^2}$$ **(1/2)**